Starter

Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

$4(k+1)^2 - 4(2k^2 - 7)$	M1		"$b^2 - 4ac$" in terms of k (either term correct)
$4k^2 - 8k - 32 = 0$ or $k^2 - 2k - 8 = 0$	A1		$b^2 - 4ac = 0$ correct quadratic equation in k
$(k-4)(k+2) = 0$	m1		Attempt to factorise, solve equation
$k = -2$, $k = 4$	A1	4	SC B1, B1 for -2 , 4 (if M0 scored)

R Forces and Newton’s laws

R4	Understand and use Newton’s third law; equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors); application to problems involving smooth pulleys and connected particles; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.

Assessed at AS and A-level

Teaching guidance

Connected particles

Students should:

- understand that usually strings will be modelled as light and inextensible
- understand that usually pulleys will be modelled as light and smooth
- understand that usually pegs will be modelled as smooth.

R

Forces and Newton's laws

R4

Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors); application to problems involving smooth pulleys and connected particles; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces.

Notes

- Questions can be set involving objects that can be modelled as particles and are connected by a light, inextensible string.
- Questions can be set that involve contexts such as a car towing a trailer or several carriages connected together as a train.
- At AS, questions will be restricted to connected particles that move horizontally or vertically. Questions involving inclined planes will **not** be set.
- When particles are connected by a string so that they do not move in the same direction, the system must **not** be treated as moving with the same acceleration and the motion of each particle must be considered separately.

8.4 Systems of Forces

Problems involving connected particles often involve **smooth**, **fixed** pulleys.

The pulley is **smooth** to enable us to ignore the effects of friction, and **fixed** so we ignore any rotational motion.

This enables us to assume that the **tension** in the string is the **same** either side of the pulley.

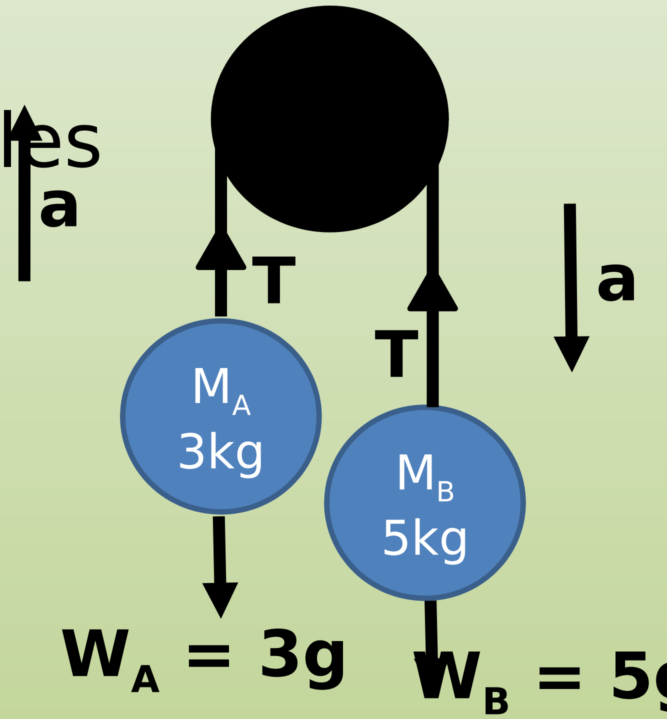
Note: we cannot treat a system involving a pulley as a single object since the

8.4 Systems of Forces

Example 1a

Particles of mass 3kg and 5kg are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is released from rest. Find:

a) The acceleration of the particles



Resolve () for A:

$T - W_A = 3a$

$T - 3g = 3a$ ❶

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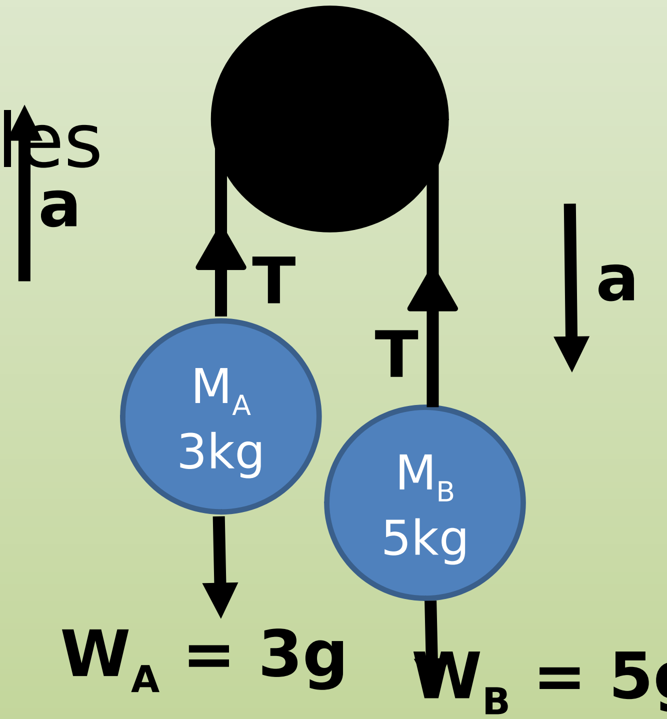


$T - 3g = 3a$ ❶

Resolve () for B:

$W_B - T = 5a$

$5g - T = 5a$ ❷

8.4 Systems of Forces

Example 1a

Particles of mass 3kg and 5kg are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is released from rest. Find:

a) The acceleration of the particles

$T - 3g = 3a$ ❶

$5g - T = 5a$ ❷

Solving simultaneously:

From ❶:

$T = 3a + 3g$ ❸

Sub in ❷:

$5g - (3a + 3g) = 5a$

$5g - 3a - 3g = 5a$

$2g = 8a$

$= \mathbf{2.45\ m/s^{-2}}$

8.4 Systems of Forces

Example 1b

Particles of mass 3kg and 5kg are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is released from rest. Find:

b) The tension in the string

$T - 3g = 3a$ ❶

$5g - T = 5a$ ❷

$T = 3a + 3g$ ❸

$T = 3a + 3g$

$T = (3 \times 2.45) + (3 \times 9.8)$

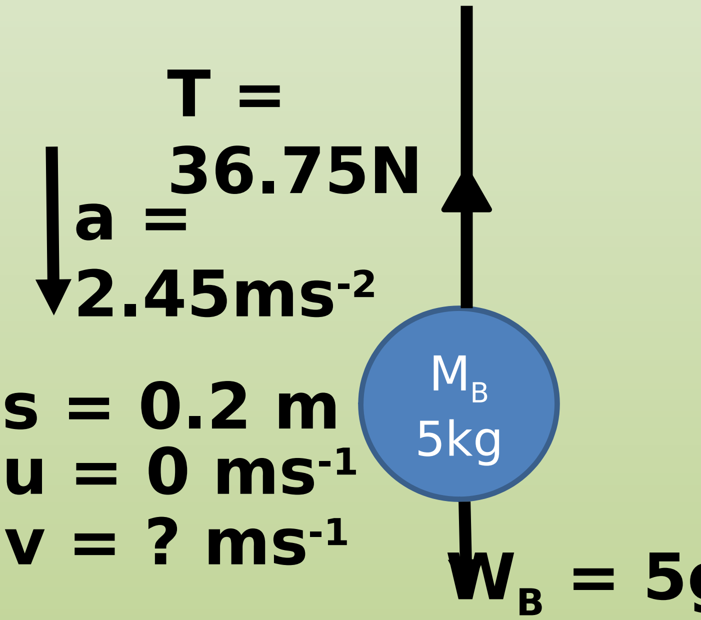
$\therefore$ **$T = 36.75$ N**

8.4 Systems of Forces

<u>Example 1c</u>

When the particles are released from rest, the 3kg particle is 0.6m above the ground, whilst the 5kg particle is 0.2m above the ground. Find:

c) The speed at which the 5kg particle hits the ground



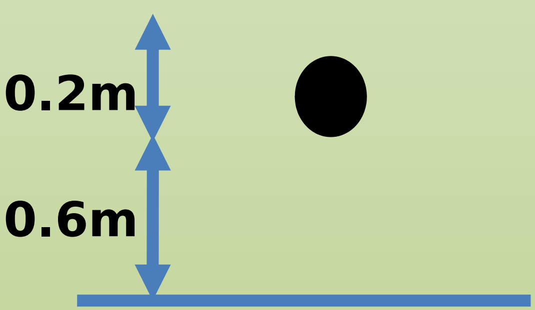
8.4 Systems of Forces

Example 1d

When the particles are released from rest, the 3kg particle is 0.6m above the ground, whilst the 5kg particle is 0.2m above the ground. Find:

d) The greatest height, above the ground, reached by the 3kg particle, assume it does not reach the pulley.

Consider particle A: it starts 0.6m off the ground.
Particle B started 0.2m off the ground.
When particle B hit the ground, particle A must have been pulled a



8.4 Systems of Forces

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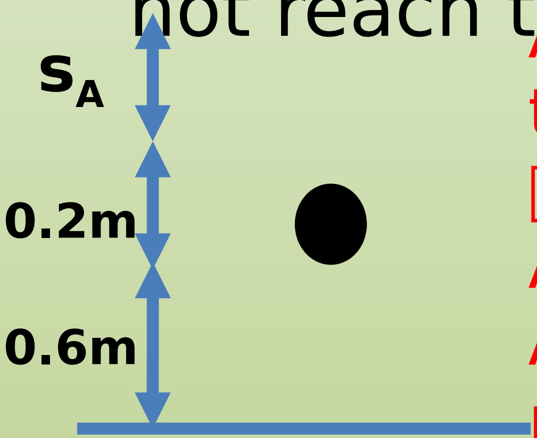

At this point, the string goes slack and the particle moves freely under gravity
 a projectile ($a = -9.8ms^{-2}$)
At it’s greatest height, $v = 0ms^{-1}$
As particle B hit the ground at ms^{-1}, particle A was launched at the same speed, so u =

8.4 Systems of Forces

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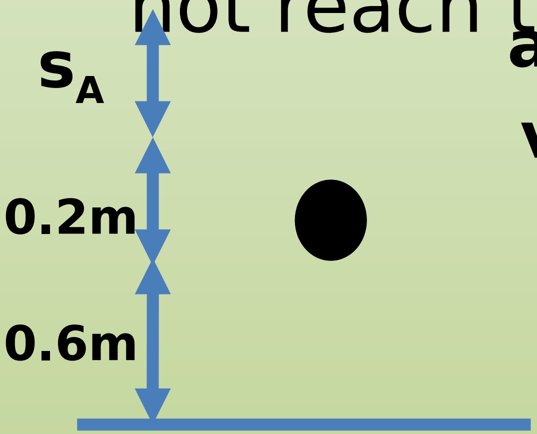

a = -9.8

v = 0

u =

8.4 Systems of Forces

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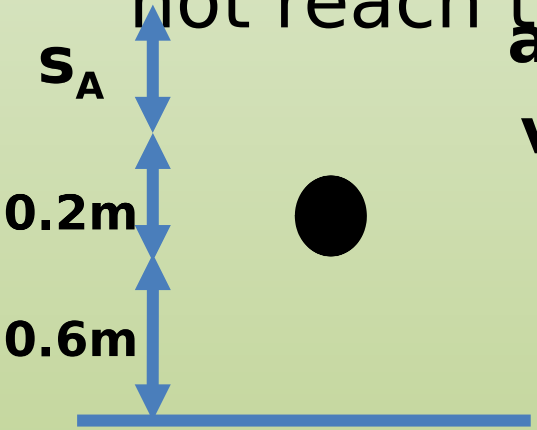


a = -9.8

v = 0

u =

8.4 Systems of Forces

You try: Example 2a

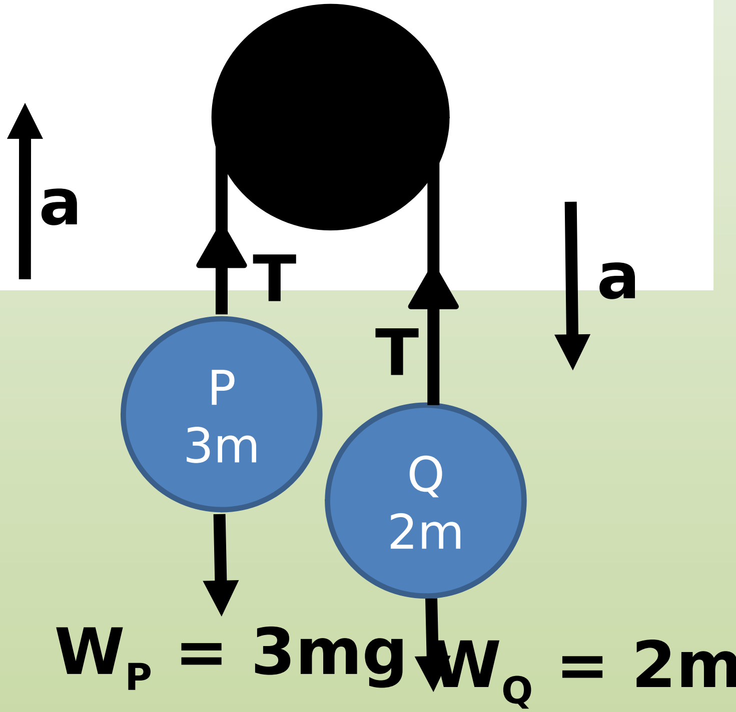
Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

a **i** Write down an equation of motion for P.

ii Write down an equation of motion for Q.

b Find the acceleration of each mass.

c Find the tension in the string.



i For P, R(↑): $T - 2mg = 2ma$ (1)

ii For Q, R(↓): $3mg - T = 3ma$ (2)

8.4 Systems of Forces

You try: Example 2b

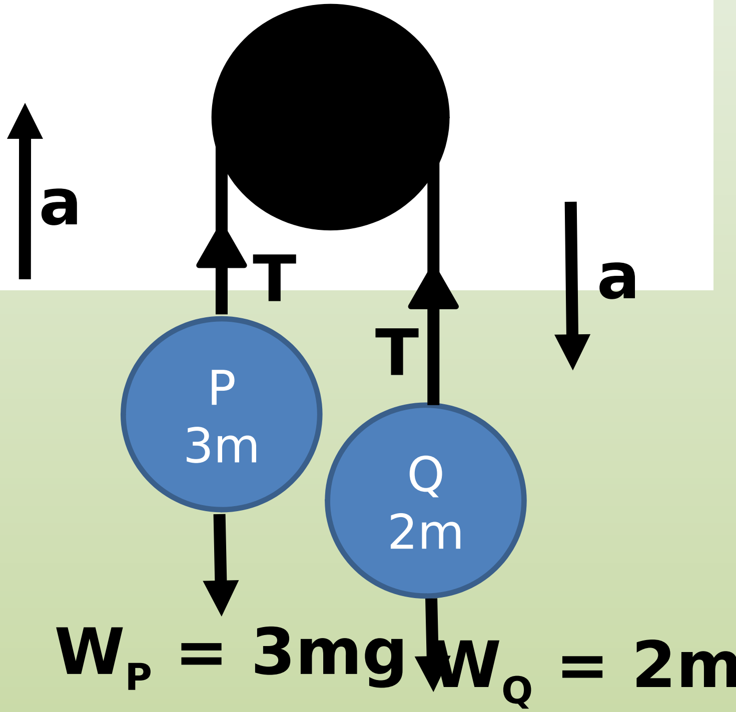
Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

a **i** Write down an equation of motion for P.

ii Write down an equation of motion for Q.

b Find the acceleration of each mass.

c Find the tension in the string.



b Adding equations (1) and (2):

$$3mg - \cancel{T} + \cancel{T} - 2mg = 3ma + 2ma$$

$$\cancel{m}g = 5\cancel{m}a$$

$$\tfrac{1}{5}g = a$$

The acceleration of each mass is $\tfrac{1}{5}g$.

8.4 Systems of Forces

You try: Example 2c

Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

a **i** Write down an equation of motion for P.

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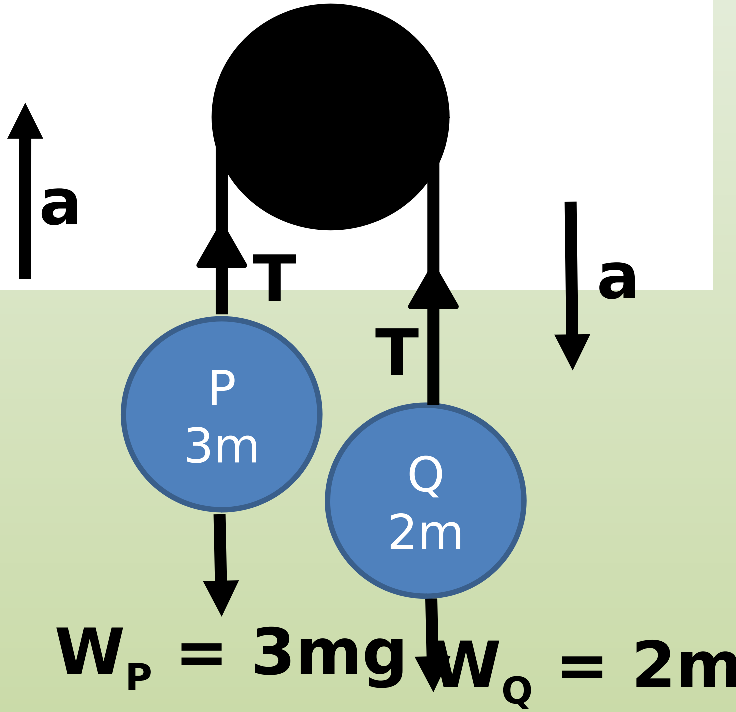
b Find the acceleration of each mass.

c Find the tension in the string.



c From (1): $T - 2mg = 2m \times \frac{1}{5}g$

$$T = \frac{12mg}{5}\text{ N}$$

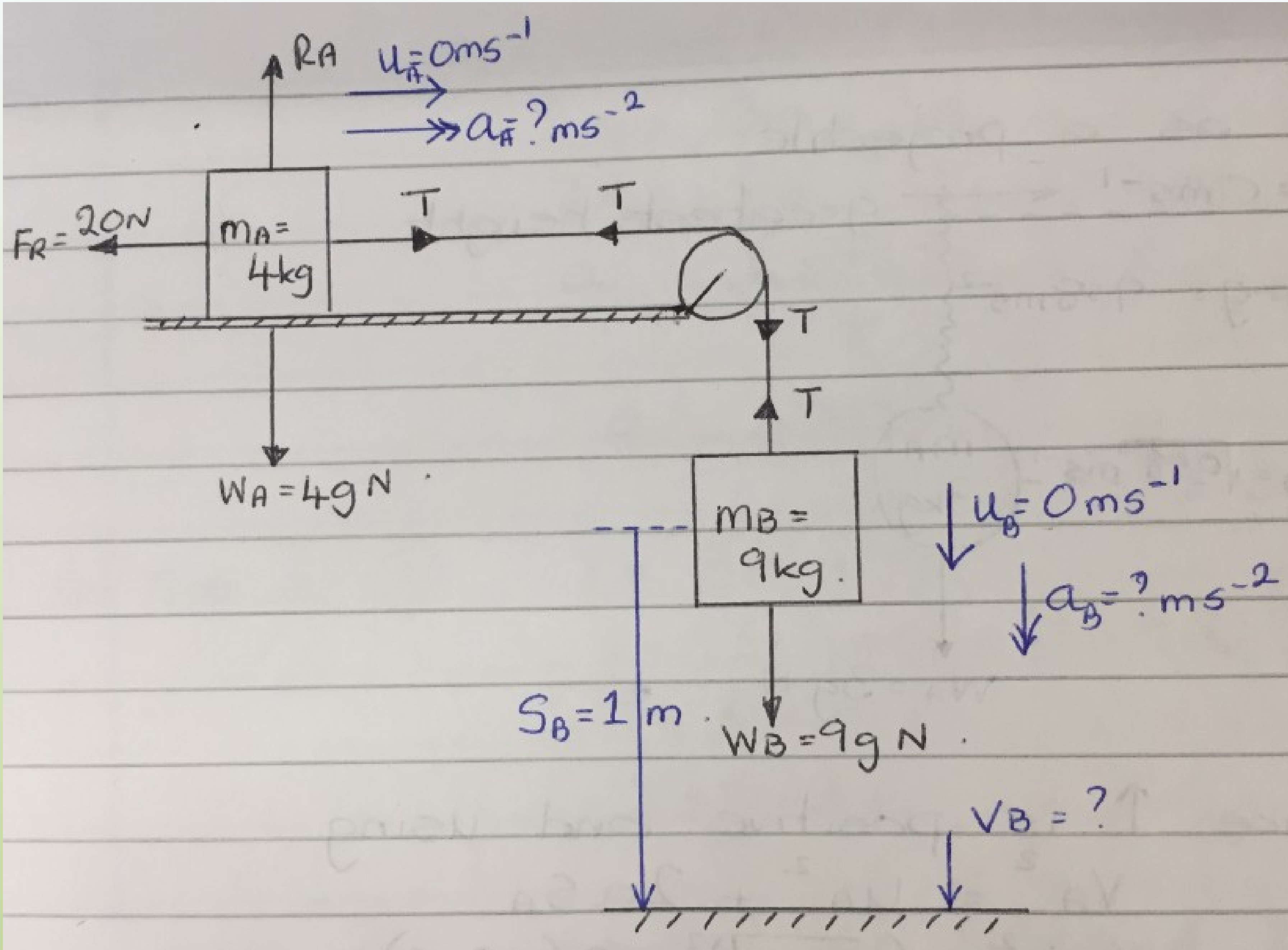
The tension in the string is $\frac{12mg}{5}$ N.

8.4 Systems of Forces

Example 3a

A block of mass 4kg rests on a rough horizontal table, with a frictional force of 20N. It is attached by a light inextensible string to a particle of mass 9kg. The string passes over a smooth pulley at the end of the table and the 9kg mass hangs freely 1m above the ground. Find:

a) The acceleration of the two masses and the tension in the string


RA
uA = 0ms⁻¹
aA = ? ms⁻²
FR = 20N
mA = 4kg
T
T
T
T
WA = 4g N
mB = 9kg
uB = 0 ms⁻¹
aB = ? ms⁻²
SB = 1 m
WB = 9g N
VB = ?

8.4 Systems of Forces

Example 3a

a) Find the acceleration of the two masses and the tension in the string

Resolve () for A:

$T - Fr = 4a$

$T - 20 = 4a$ ❶

Resolve () for B:

$W_B - T = 9a$

$9g - T = 9a$ ❷

8.4 Systems of Forces

<u>Example 3a</u>

a) Find the acceleration of the two masses and the tension in the string

$T - 20 = 4a$ ❶

$9g - T = 9a$ ❷

❶ + ❷: $9g - 20 = 9a + 4a$

$9g - 20 = 13a$

Sub into ❶:

8.4 Systems of Forces

<u>Example 3b</u>

b) The speed with which the 9kg mass hits the ground

We know:
It started from rest $u = 0$ m/s
It was 1m off the ground $s = 1$m

We now know:
The acceleration $a =$ m/s^2

We want to know:
$v = ?$

8.4 Systems of Forces

Example 3c

c) Work out how far the 4kg mass travels before it comes to rest, assuming it does not reach the pulley

The 4kg mass is particle A. The 9kg mass is particle B.

Two distinct stages: Stage 1 – before B hits the ground

Stage 2 – after B hits the ground

During stage 1, A will travel 1m (because B travelled 1m).

8.4 Systems of Forces

Example 3c

c) Work out how far the 4kg mass travels before it comes to rest, assuming it does not reach the pulley

During stage 2, A's acceleration will change as there will no longer be any tension in the string.

Resolve ():

$-Fr = 4a$

$-20 = 4a$

$\mathbf{a = -5ms^{-2}}$

8.4 Systems of Forces

Example 3c

c) Work out how far the 4kg mass travels before it comes to rest, assuming it does not reach the pulley

During stage 2, we know:

$a = -5ms^{-2}$

$u =$ (the same speed that B hit the ground)

$v = 0$

We want to know:

$s = ?$

8.4 Systems of Forces

Example 3c

c) Work out how far the 4kg mass travels before it comes to rest, assuming it does not reach the pulley

During stage 2, we know:

$a = -5ms^{-2}$

$u =$ (the same speed that B hit the ground)

$v = 0$

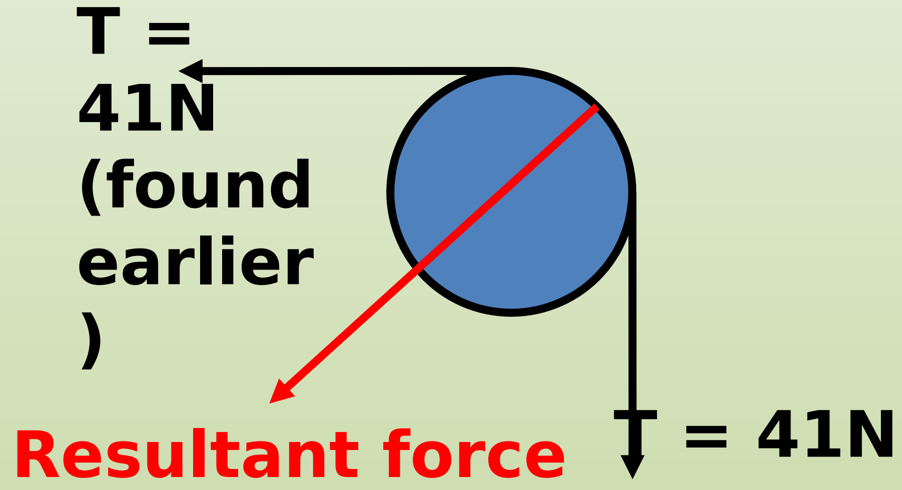
We want to know:

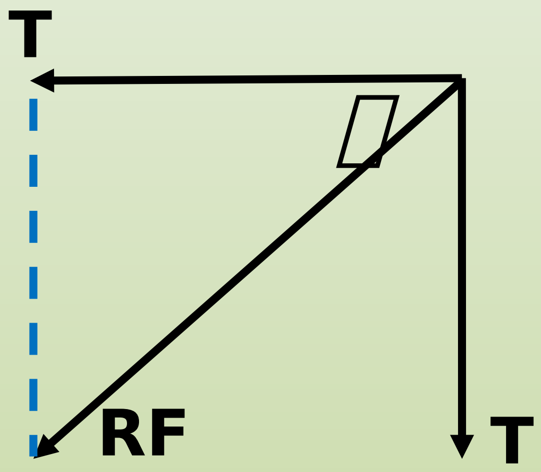
$s = ?$

8.4 Systems of Forces

Example 3d

d) Work out the resultant force acting on the pulley before the 9kg mass hits the ground



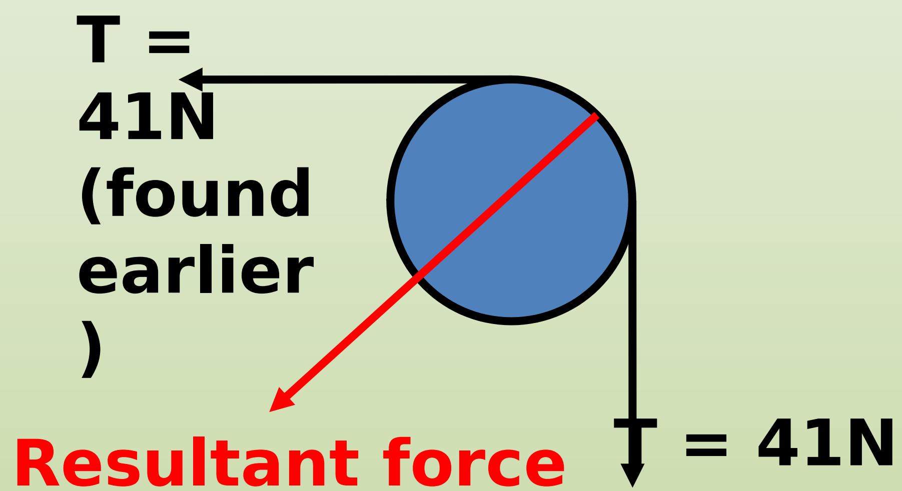


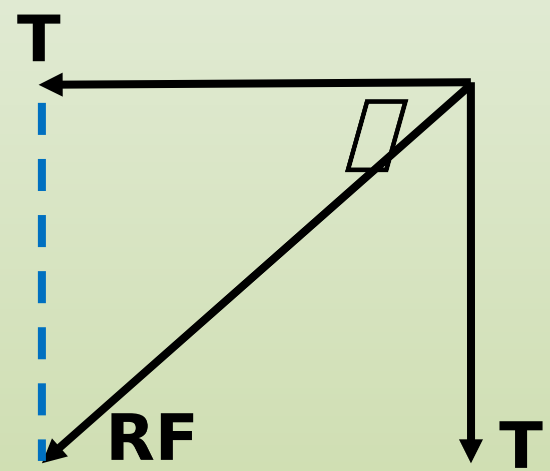
Magnitude of resultant force =

8.4 Systems of Forces

Example 3d

d) Work out the resultant force acting on the pulley before the 9kg mass hits the ground





Resultant force is 58N acting at 45 degrees below

8.4 Systems of Forces

You try: Example 4a

Two particles A and B of masses 0.4 kg and 0.8 kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5 m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5 m above horizontal ground. A frictional force of magnitude $0.08g$ opposes the motion of particle A. The system is released from rest. Find:

a the acceleration of the system

b the time taken for B to reach the ground

c the total distance travelled by A before it first comes to rest.

For A only: $R(\rightarrow)$, $T - 0.08g = 0.4a$ (1)

For B only: $R(\downarrow)$, $0.8g - T = 0.8a$ (2)

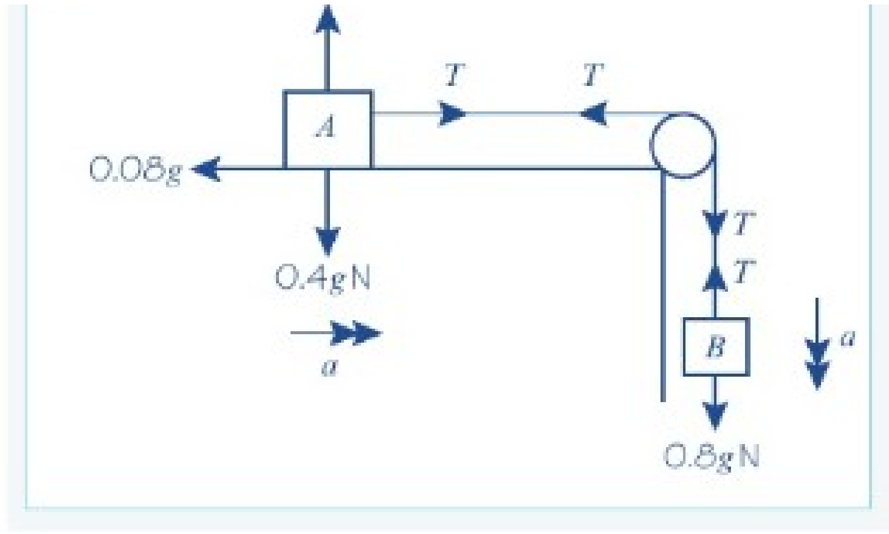
Add (1) and (2):

$$0.8g - \cancel{T} + \cancel{T} - 0.08g = 0.8a + 0.4a$$

$$0.72g = 1.2a$$

$$0.6g = a$$

The acceleration of the system is $0.6g$



8.4 Systems of Forces

You try: Example 4b

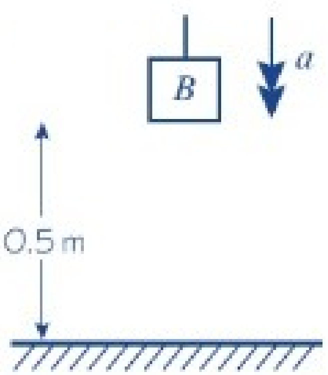
Two particles A and B of masses 0.4 kg and 0.8 kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5 m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5 m above horizontal ground. A frictional force of magnitude $0.08g$ opposes the motion of particle A. The system is released from rest. Find:

a the acceleration of the system

b the time taken for B to reach the ground

c the total distance travelled by A before it first comes to rest.

b



$u = 0,\ s = 0.5,$

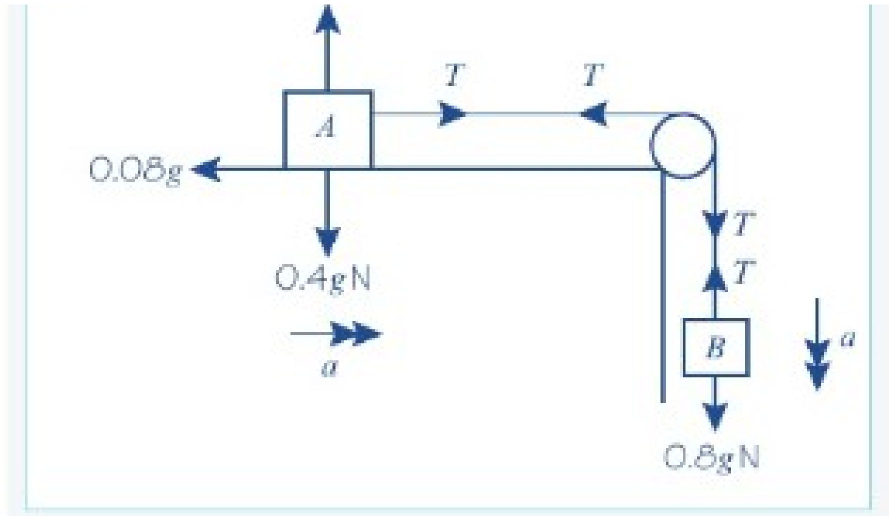
$a = 5.88,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$0.5 = 0 + \frac{1}{2} \times 5.88 \times t^2$

$t = 0.41$ (2 s.f.)

The time taken for B to hit the ground is 0.41 s (2 s.f.)



8.4 Systems of Forces

You try: Example 4c

Two particles A and B of masses 0.4 kg and 0.8 kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5 m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5 m above horizontal ground. A frictional force of magnitude $0.08g$ opposes the motion of particle A. The system is released from rest. Find:

a the acceleration of the system

b the time taken for B to reach the ground

c the total distance travelled by A before it first comes to rest.

c Find the speed of B when it hits the ground.

$u = 0,\ a = 5.88,\ t = 0.412\,39,\ v = ?$

$v = u + at$

$v_B = 0 + 5.88 \times 0.412\,39 = 2.424\,87\ \text{m s}^{-1}$.

Speed of A on the table is $2.424\,87\ \text{m s}^{-1}$.

Once B hits the ground the string will go slack and A will begin to decelerate as it slides against the friction on the table.

From (1): $-0.08g = 0.4a'$

$a' = -0.2g$

$u_A = 2.424\,87,\ v = 0,\ a' = -0.2g,\ s = ?$

$v^2 = u^2 + 2as$

$0^2 = 2.424\,87^2 - 0.4gs$

$s = 1.5\ \text{m}$ (2 s.f.)

A slides a further 1.5 m along the table before it comes to rest.

$\therefore$ Total distance moved by A is

$0.5 + 1.5 = 2.0\ \text{m}$ (2 s.f.)